Sound Lens

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1 Introduction

In the 19th century, a device was invented for the purpose of focusing sound waves. It consisted of a balloon, the same shape as a converging lens, filled with a gas, such that the velocity of sound in the balloon was different from the velocity in air.

Find the focal length of such a lens, having radii of curvature of 2m, filled with N_2O, assuming the gases to be ideal and the temperature inside the balloon to be the same as the surrounding air. Neglect any effect from the balloon material.

The effective molecular mass of air can be taken as 29 and the relative molecular mass of N_2O is 44. The ratio of principal specific heats, γ, is 1.40 for diatomic and 1.33 for triatomic gases.

2 Speed of Sound

Since the temperature is not specified, it is not possible to determine the speed of sound in either of the gases. However, a ratio of the two can be determined.

The formula for the speed of sound in a gas is given by:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Since air is almost completely composed of diatomic gases, γ_{air} can be taken as 1.40, and $\gamma_{nitrous}$ as 1.33.

$$v_{air} = \sqrt{\frac{1.40 * RT}{29}}$$

$$v_{nitrous} = \sqrt{\frac{1.33 * RT}{44}}$$

$$\frac{v_{air}}{v_{nitrous}} = \sqrt{\frac{1.40 * RT * 44}{1.33 * RT * 29}}$$

$$\frac{v_{air}}{v_{nitrous}} = \sqrt{\frac{140 * 44}{133 * 29}}$$

$$\frac{v_{air}}{v_{nitrous}} = \sqrt{\frac{6160}{3857}} \qquad (1)$$

3 Lens Makers Equation

The following formula is the lensmakers equation, where n_1 is the refractive index of the material outside the lens, and n_2 is the refractive index of the material inside the lens:

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Now recall that for light, the refractive index, n, is given by:

$$n = \frac{c}{v}$$

Where c is the speed of light in a vacuum and v is the speed of light through some medium. However, sound waves cannot travel in a vacuum, so a similar definition does not make much sense for sound, but thanks to the the division of one refractive index by another in the lens makers equation, the speed of light terms cancel away and leave a simple ratio of velocities:

$$\frac{n_2}{n_1} = \frac{c * v_1}{c * v_2}$$
$$\frac{n_2}{n_1} = \frac{v_1}{v_2}$$
$$\frac{n_{nitrous}}{n_{air}} = \frac{v_{air}}{v_{nitrous}}$$

Recall equation 1 from section 2, where this is exactly what was calculated:

$$\frac{v_{air}}{v_{nitrous}} = \sqrt{\frac{6160}{3857}}$$

Using this, the focal length can be calculated:

$$\frac{1}{f} = \left(\sqrt{\frac{6160}{3857}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f} = \left(\sqrt{\frac{6160}{3857}} - 1\right)\left(\frac{1}{2} - \frac{1}{-2}\right)$$
$$\frac{1}{f} = \left(\sqrt{\frac{6160}{3857}} - 1\right)\left(\frac{1}{2} + \frac{1}{2}\right)$$
$$\frac{1}{f} = \left(\sqrt{1.5971} - 1\right) * 1$$
$$\frac{1}{f} = 1.26 - 1$$
$$\frac{1}{f} = 0.26$$
$$f = 3.8\ m$$

Therefore the focal length of this particular sound lens is 3.8 metres.